Universität
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Optimal Cycling Strategies for two Cooperating Riders

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Science & Cycling
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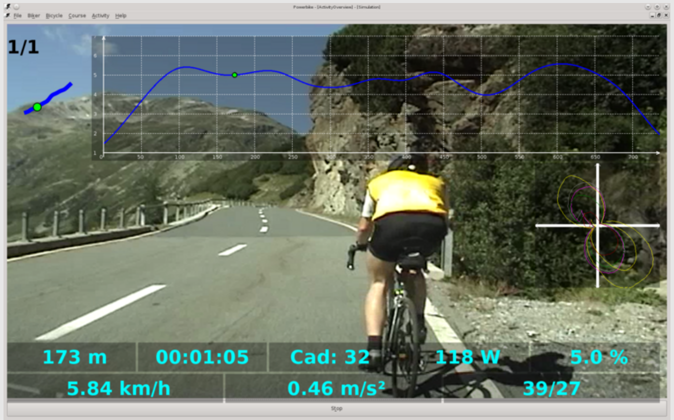
Powerbike Project

Goals

- Acquisition, analysis, and visualization of performance parameters in lab and field
- Realistic simulation of road cycling on real courses
- Optimization of pacing strategies
- Modelling of physiological parameters

Simulator



Pacing Optimization

How to improve race performance?

- Pre-race improvements: Endurance training, Riding technique, Bike technology
- Pacing strategy

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Strategies in practice

- Depend mainly on the expertise of the athlete and the trainer
- Common ground on simple courses

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- Get an objective view on strategies.
- Use mathematical models and optimization.

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- Get an objective view on strategies.
- Use mathematical models and optimization.

Strategies based on mathematical models

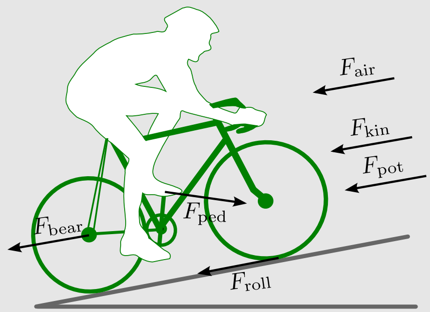
- Available for individual time trials
- Only few works with several riders

Model for Bicycle Mechanics

Equilibrium of forces

$$F_{\text{resist}} = \underbrace{mg\frac{\mathrm{d}h}{\mathrm{d}x}}_{F_{\text{pot}}} + \underbrace{\mu mg}_{F_{\text{roll}}} + \underbrace{(\beta_0 + \beta_1 v)}_{F_{\text{bear}}} + \underbrace{\left(m + \frac{I_w}{r_w^2}\right)\dot{v}}_{F_{\text{kin}}} + \underbrace{\frac{1}{2}c_d \rho A v^2}_{F_{\text{air}}} = \frac{\eta}{\gamma}\frac{l_c}{r_w}F_{\text{ped}}$$

Cyclist and bicycle		Course and environment	
total mass (cyclist, bike)	m	friction factor	μ
wheel circumference	c_w	gravity factor	g
wheel radius	r_w	drag coefficient	c_d
wheel inertia	I_w	air density	ρ
cross-sectional area	A	height	$h(x)$
length of crank	l_c	chain efficiency	η
bearing coefficient	β_0	mechanical gear ratio, bicycle	γ
bearing coefficient	β_1		



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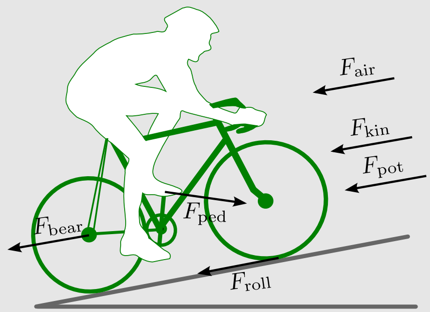
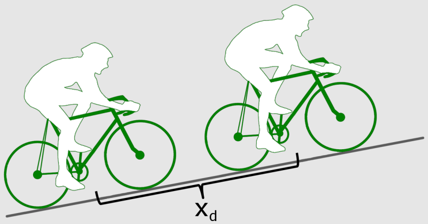


2-rider extension

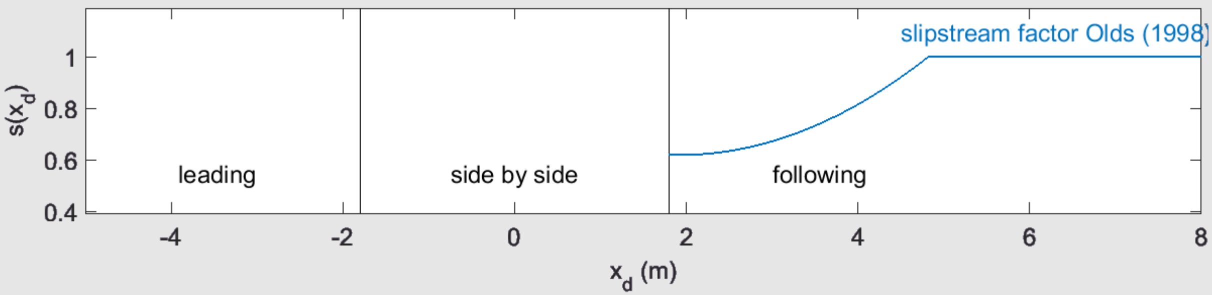
Slipstream reduces air resistance

Model for Slipstream

Slipstream factor



Wind resistance: $F'_{\text{air}} = s(x_d)F_{\text{air}}$
Length of the bike (l): 1.8m
Minimum tire distance (d_{min}): 0.1m
Reduction in sweet spot (γ): 37%



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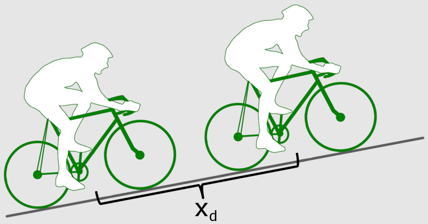

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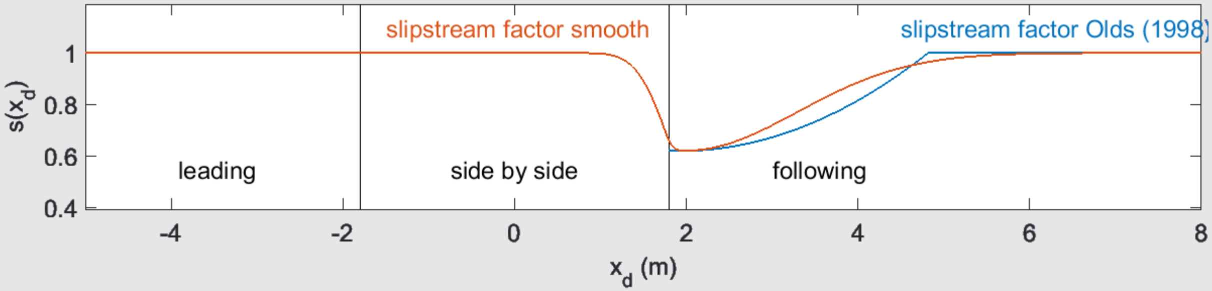

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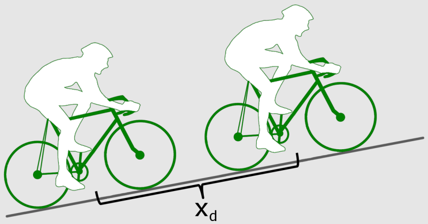

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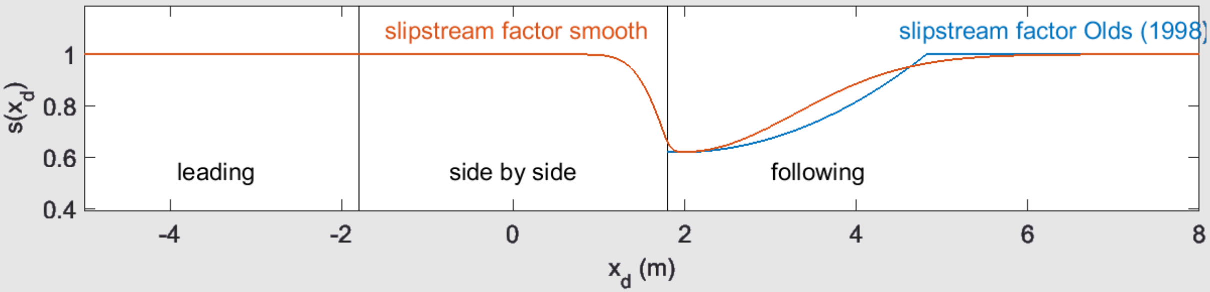


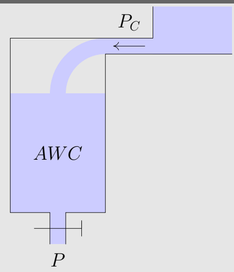
Formula

$$s(x_d) = 1 - \gamma \exp(-\alpha(x_d)(x_d - (l + d_{\text{min}}) - 0.1)^2)$$
$$\alpha(x_d) = -\frac{6-0.3}{2} \tanh(\epsilon(x_d - (l + d_{\text{min}}) + 0.1)) + \frac{6-0.3}{2}$$

Physiological Model

Critical power model

- Critical power P_C
- Anaerobic work capacity AWC
- Change in awc level: $a\dot{w}c(t) = P_C - P(t)$



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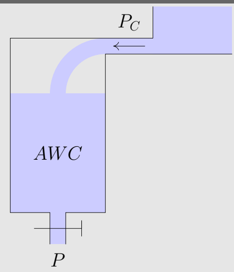

Morton's 3-parameter critical power model

- Maximum instantaneous power $P_{M,inst}$
- Change in awc level: $\dot{awc}(t) = (P_{M,inst} - P_C)\frac{P_C - P(t)}{P_{M,inst} - P(t)}$

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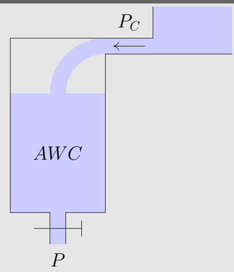

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Maximum power constraint

- Maximum experiment power $P_{M,exp}$
- Maximum power constraint: $P(t) \leq P_C + (P_{M,exp} - P_C)\frac{awc(t)}{AWC} =: P_m(t, awc(t))$

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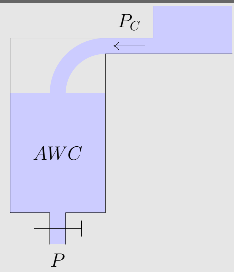

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2-rider extension

depletion rate diminishes if AWC is full

Optimization

Optimal control problem

Minimize the cost-function based on controls $P_1(t)$ and $P_2(t)$

$$J = T$$

subject to the dynamic constraints, boundary conditions

$$\begin{aligned}
\dot{x}_1(t) &= v_1(t) & x_1(0) &= x_{1,0} & x_1(T) &= x_{1,f} \\
\dot{x}_d(t) &= v_2(t) - v_1(t) & x_d(0) &= x_{d,0} & x_d(T) &\geq 0 \\
\dot{v}_i(t) &= F_{\text{mech}}(v_i(t), x_1(t), x_d(t), P_i(t)) & v_i(0) &= v_{i,0} & & \\
a\dot{w}c_i(t) &= F_{\text{phys}}(P_i(t)) & awc_i(0) &= AWC_{i,0} & &
\end{aligned}$$

and the path constraints

$$\begin{aligned}
0 \leq &\ awc_i(t) \\
0 \leq &\ P_i(t) \leq P_{m,i}(t, awc_i(t))
\end{aligned}$$

Numerical solver

- GPOPS II (RP Optimization Research LLC)
- SNOPT (Stanford Business Software Inc.)

Example 1

Riders parameters

parameter	rider 1	rider 2
P_C	300 W	300 W
AWC	25000 J	25000 J
$P_{M,inst}$	15000 W	15000 W
$P_{M,exp}$	1100 W	1100 W
Weight	80 kg	80 kg

Track parameters

- Length $x_f = 5$ km
- Constant slope of 0 %

Example 1

Riders parameters

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P_C	$300\,\mathrm{W}$	$300\,\mathrm{W}$
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Track parameters

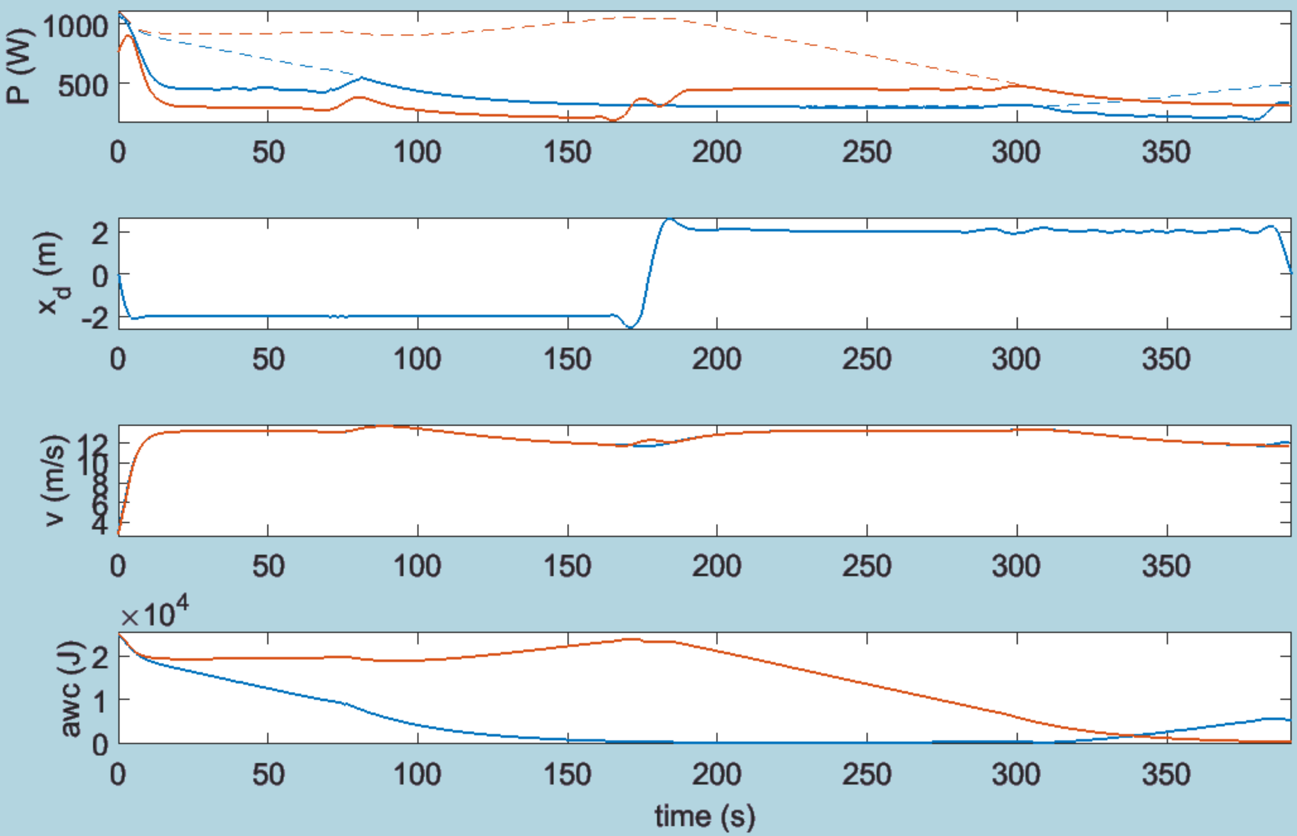
- Length $x_f = 5\,\mathrm{km}$
- Constant slope of $0\,\%$

Results

- Race-time coop: 6 min 31 sec
- Race-time rider 1: 6 min 54 sec ($+5.8\,\%$)
- Race-time rider 2: 6 min 54 sec ($+5.8\,\%$)

Example 1

Dynamics



Example 2

Riders parameters

parameter	rider 1	rider 2
P_C	300 W	350 W
AWC	25000 J	25000 J
$P_{M,inst}$	15000 W	15000 W
$P_{M,exp}$	1100 W	1100 W
Weight	80 kg	80 kg

Track parameters

- Length $x_f = 5$ km
- Constant slope of 0 %

Results

- Race-time coop: 6 min 20 sec
- Race-time rider 1: 6 min 54 sec (+8.9 %)
- Race-time rider 2: 6 min 34 sec (+3.6 %)

Summary

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- Model to simulate two riders
- Slipstream reduces air resistance by 30% in best position
- Race-time can be reduced by over 5% for equally trained riders
- Less trained athletes benefit from well trained colleagues

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Future Work

- Extend to more than two riders
- Apply to real world tracks
- Non-cooperative strategies

dvs Workshop Modelling in Endurance Sports

- University of Konstanz
- 11.9.2016 - 13.9.2016
- Invited speakers: Chris Abbiss, Jim Martin
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