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Introduction: Winning a stage of the Tour de France depends on several factors. Besides preparation and the physiological and mental condition, the racing strategy plays a major role. There have been several works that determine optimal strategies for individual time trials based on mathematical models, but only few that consider more than one competitor.

One of the first looking into strategies for two competitors was Pitcher [1]. He considered slipstream effects in running and pointed out that the runner behind can take advantage of this effect and win the race by taking over shortly before the finish line. A major limitation is that the front runner does not react to the strategy of his opponent. Dahmen [2] adapted this approach to cycling and Aftalion [3] generalized it by allowing both runners a free strategy.

In this work we propose a model for two riders cooperating in order to finish a course in the shortest possible time. We will present the underlying mathematical models, the optimal control problem and provide a numerical example.

Methods: We use a physical model describing the equilibrium of the rider's pedal force and the forces induced by aerodynamic drag, friction, gravitation and inertia (see [4]). The aerodynamic drag coefficient is additionally multiplied by the slipstream factor shown in Figure 1. The physiological capabilities of the athlete are modelled by a dynamic version of the 3-parameter critical power model of Morton [5].

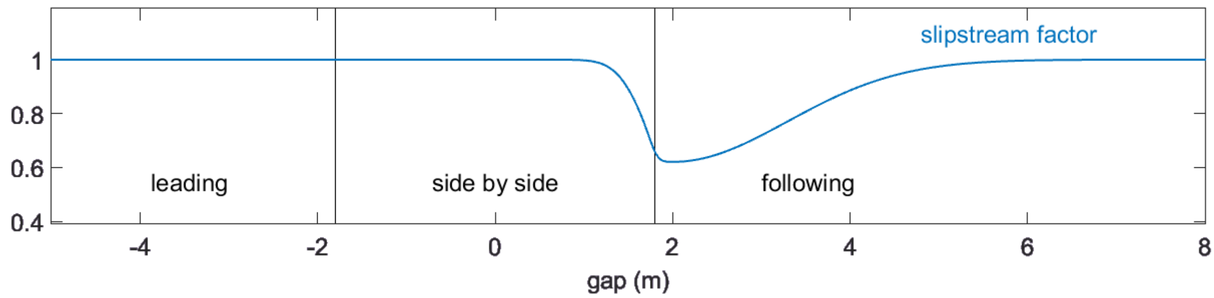

Figure 1: Slipstream factor related to the distance of the centers of gravity of the two riders.

With these models on hand we can formulate the optimal control problem: Control the power output of each rider in order to minimize the time needed to complete the course regarding the physiological capabilities of both riders. It is solved by the state-of-the-art optimal control solver GPOPS-II [6].

Results: The optimal strategy for two equally strong riders on a flat course of 2 km length is shown in Figure 2. The total race-time is 181 sec, which is a reduction of over 5% compared to the individual optimal strategy.

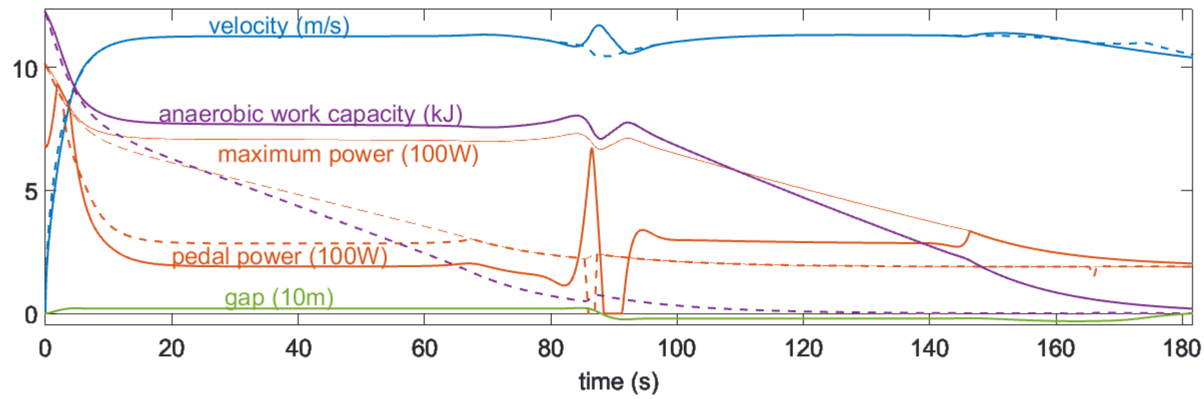

Figure 2: Optimal strategy on a flat course of 2 km length. Solid lines are associated with rider 1 and dashed lines with rider 2.

Conclusions: In races we often have the situation that several riders get ahead of the peloton. In order to stay in front, they are forced to team up. We provided a model to simulate the interactions between two riders. With a simple example we showed that a race can be finished significantly faster if they cooperate as well as when and how the overtaking maneuver should be performed.

References:

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[6] Patterson, Michael A., and Anil V. Rao. "GPOPS-II: A MATLAB software for solving multiplephase optimal control problems using hp-adaptive Gaussian quadrature collocation methods and sparse nonlinear programming." *ACM Transactions on Mathematical Software (TOMS)* 41.1 (2014): 1.