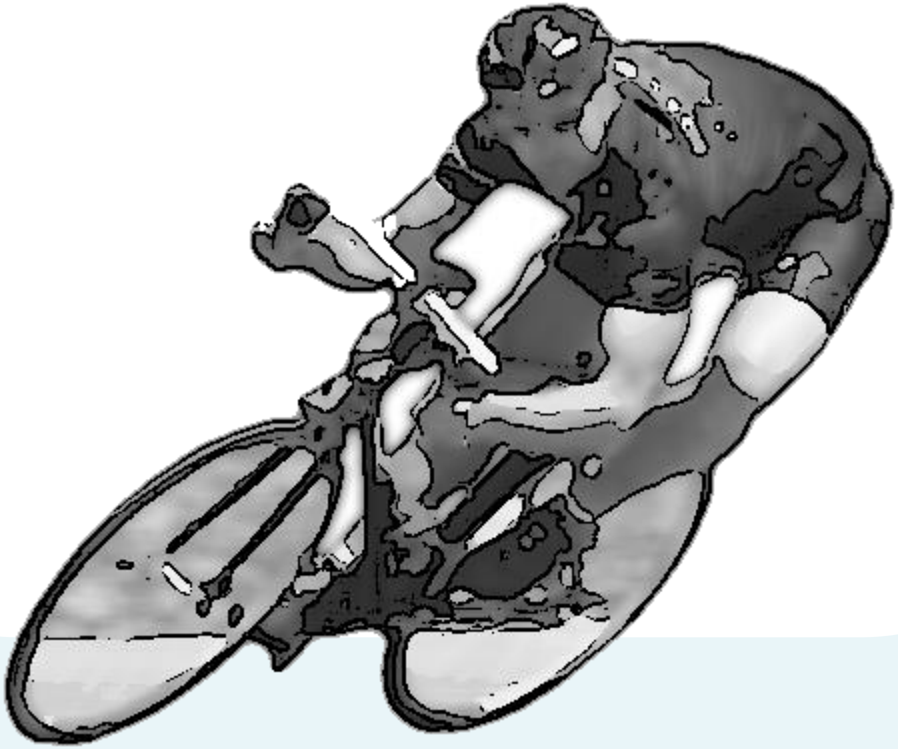
Optimal distribution of power output and braking for corners in road cycling



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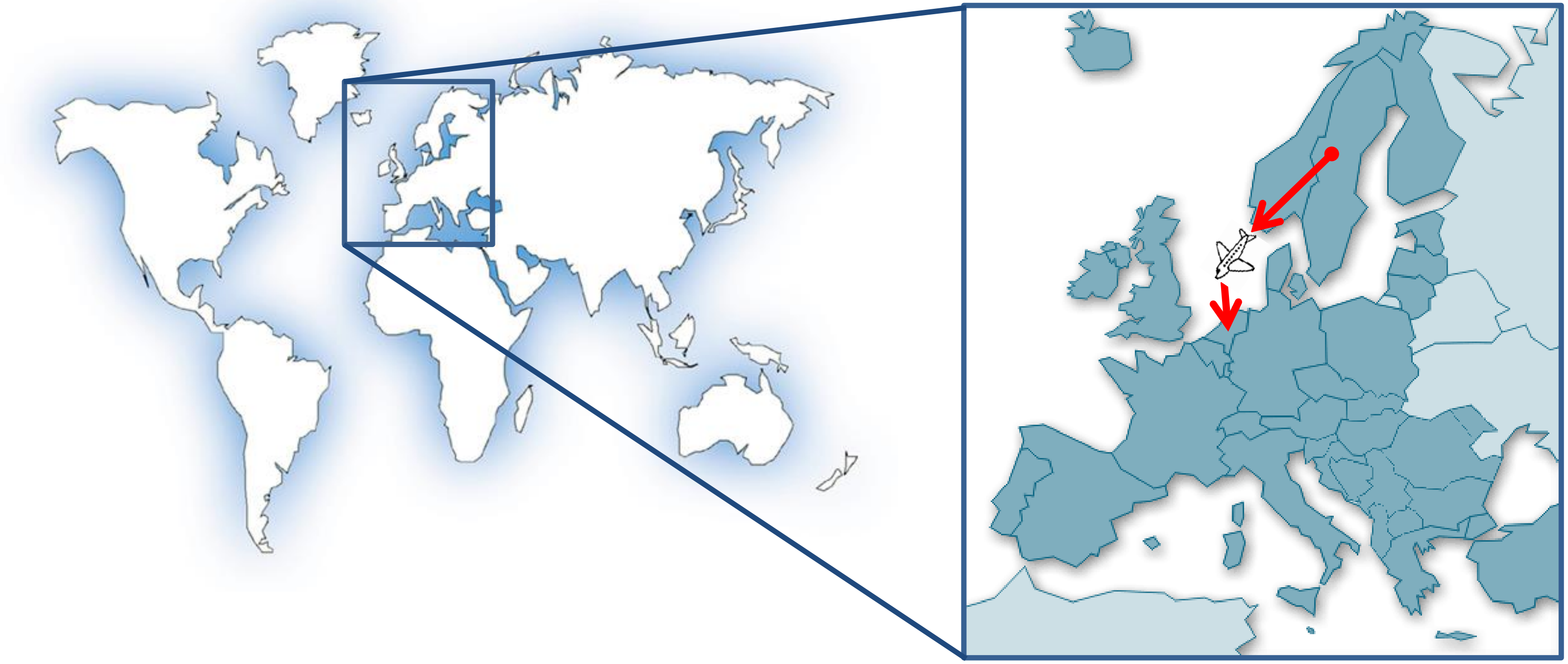
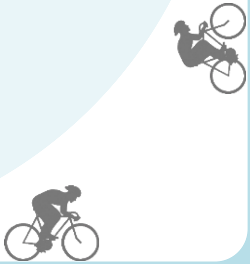


Background

The effect of course profile and ambient wind on the optimal pacing strategy has been examined in numerous studies.

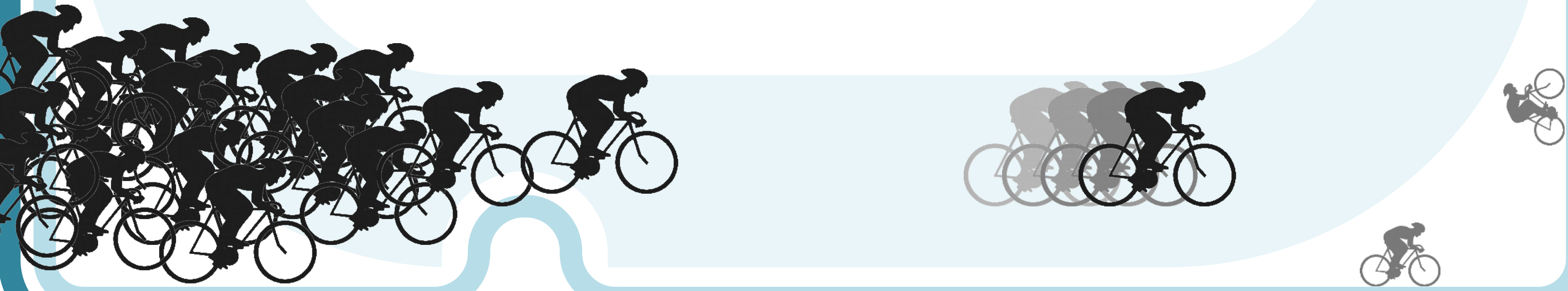
All studies on pacing strategies have been performed without considering course bends.

Sharp course bends at high speed are common in short circuit races and on mountain descents.



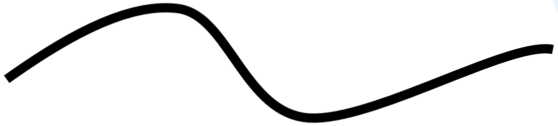
Aim

Examine the optimal power output and braking power distribution for cornering in road cycling.



Numerical model

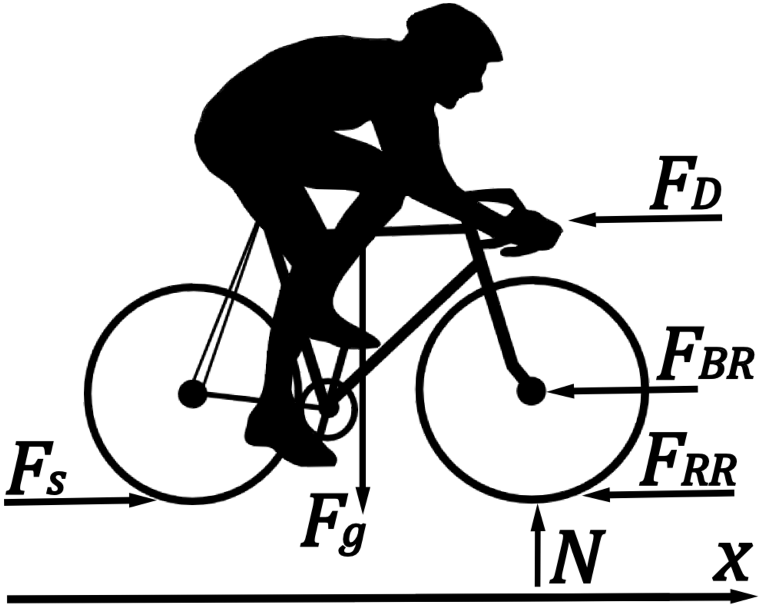
- Point-mass ●
- Course: Cubical splines



- External forces
- Motion equation
- Differential equation solver
- Optimization

Motion equation

- External forces
  - Propulsive force
  - Aerodynamic drag
  - Rolling resistance
  - Bearing resistance
  - Gravity
  - Normal force
- Inertia
- Transformation
  - Dependent variable: *t*
  - Independent variable: *x*



$$t'' = -(t')^4 \frac{P\cdot\eta_{tr}}{m_s} + (t')^3 \frac{C_{RR}\cdot m_{tot}\cdot g}{m_s} + (t')^2 \frac{b_1\cdot t'+b_2}{m_s} + t' \frac{(C_D A+A_w)\rho}{2\cdot m_s}$$

- Solution with Runge-Kutta 4

Optimization

Minimize: $T = \sum_{i=1}^{K} \Delta t_i$

Subject to: $0 < AL_i \leq AL_{max}$

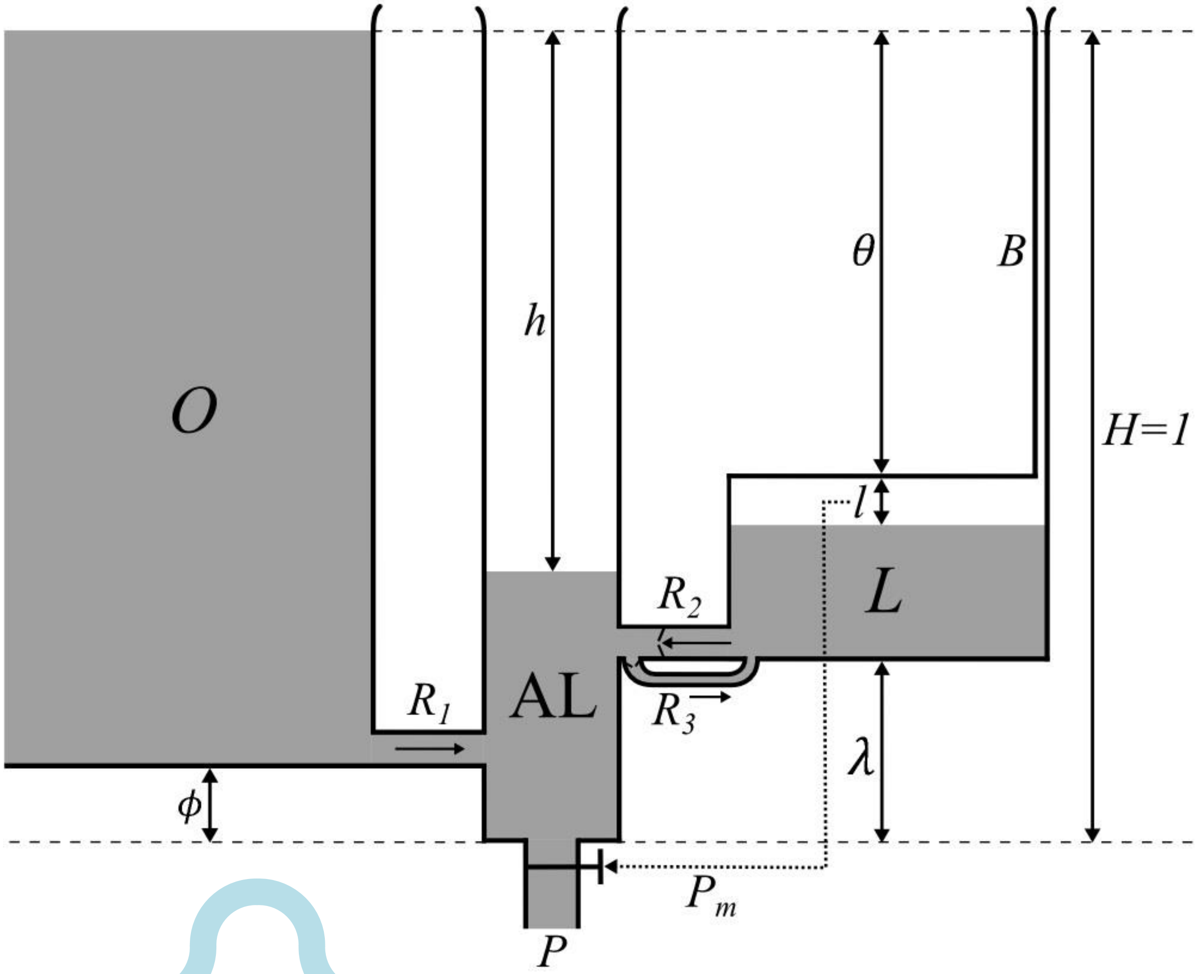
$$0 < L_i \leq L_{max}$$

$$P_{min} \leq P_i \leq P_{max_i}$$

$$\sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \frac{\left(\frac{dx}{dt}\right)^4}{r^2}} \leq \mu g$$

Constraints

Margaria-Morton model



Optimization

Minimize: $T = \sum_{i=1}^{K} \Delta t_i$

Subject to:

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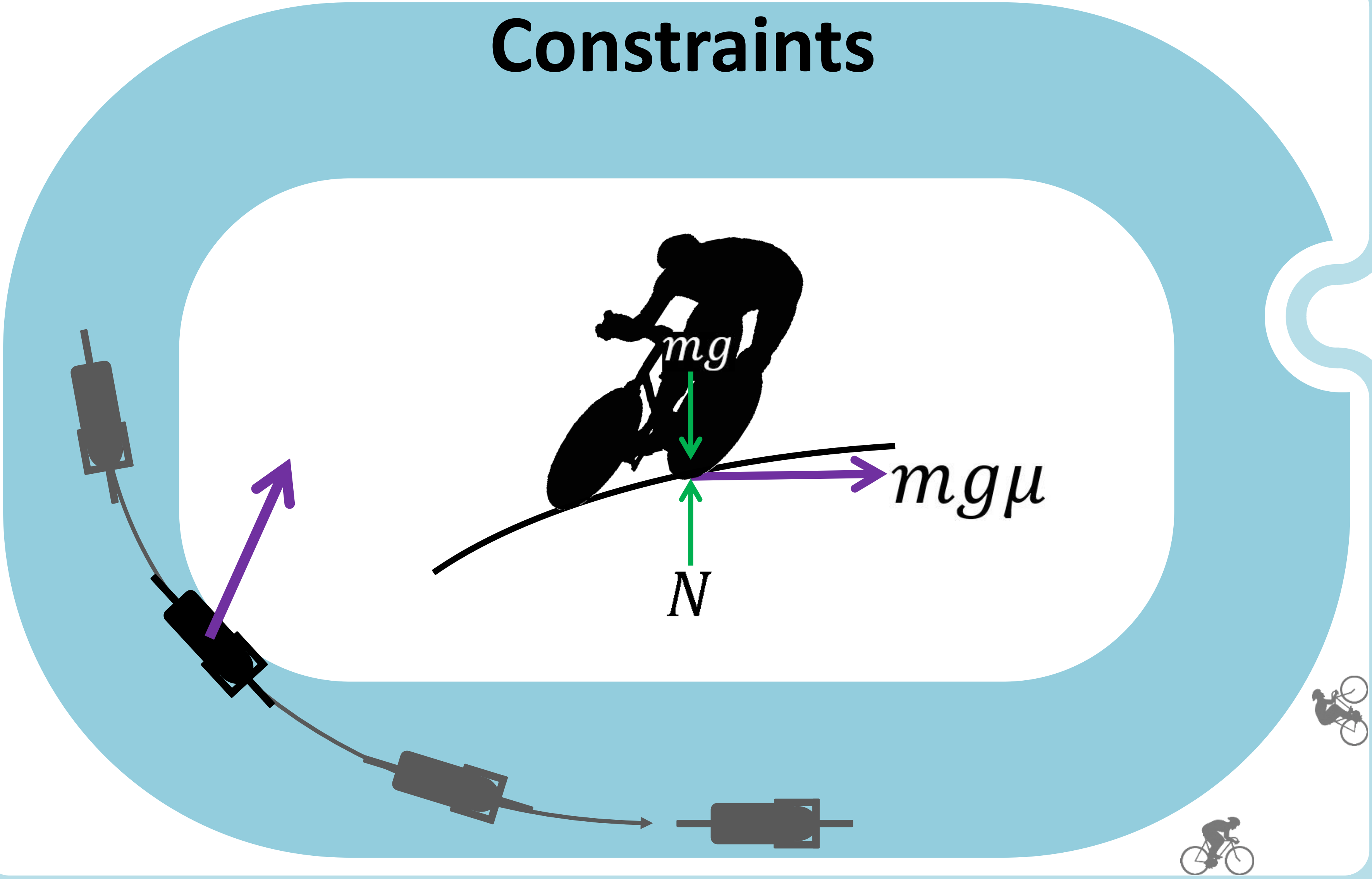
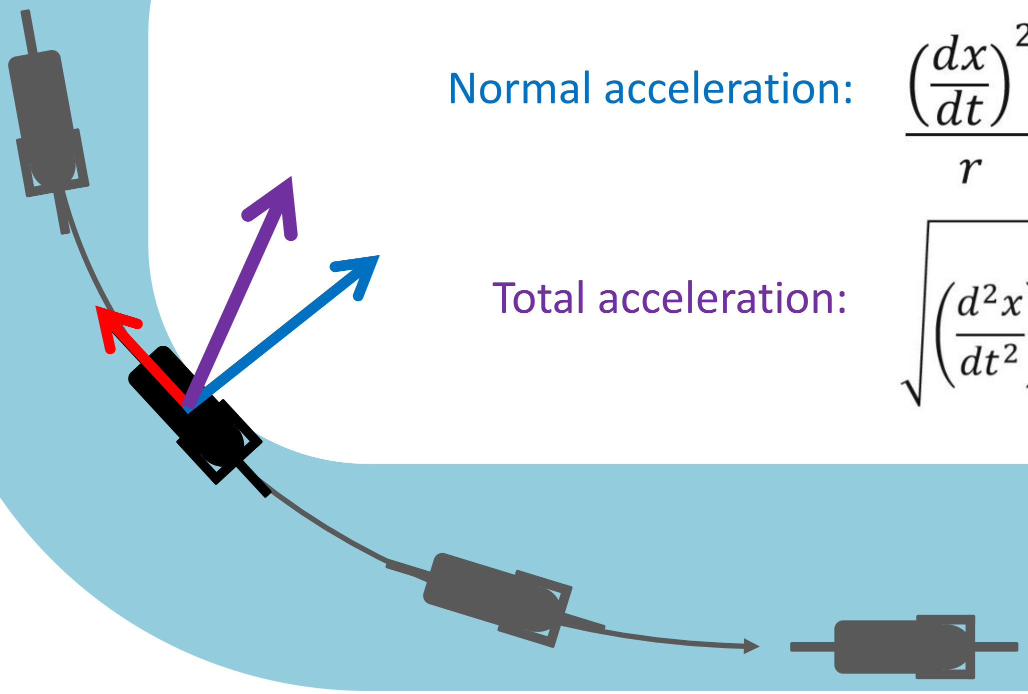


Constraints

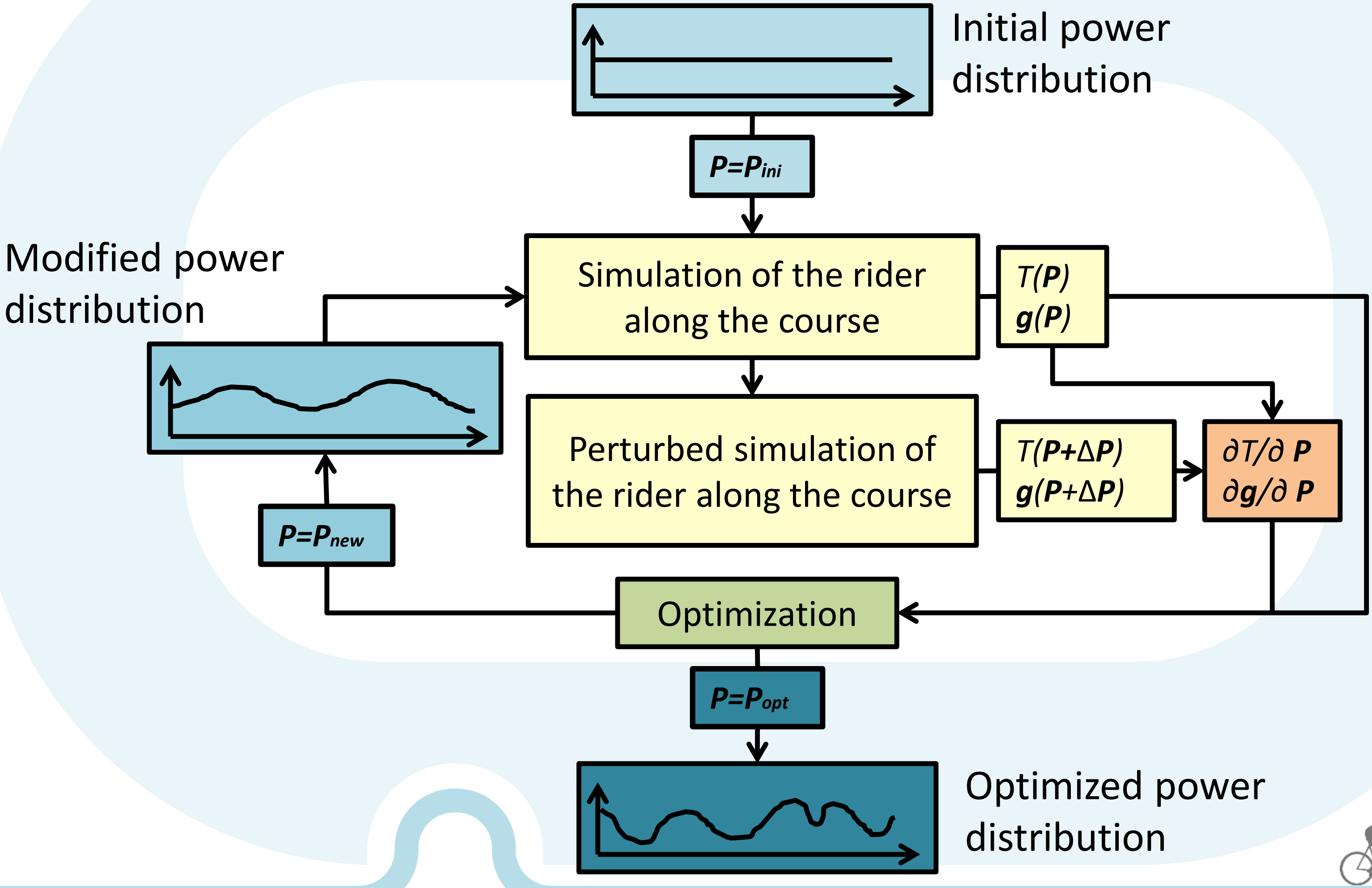


Tangential acceleration: $\frac{d^2x}{dt^2}$ (Braking)

Normal acceleration: $\frac{\left(\frac{dx}{dt}\right)^2}{r}$ (Turning)

Total acceleration: $\sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \frac{\left(\frac{dx}{dt}\right)^4}{r^2}} \le \mu g$

Optimization



Initial power distribution

$P=P_{ini}$

Simulation of the rider along the course

$T(\mathbf{P})$
$g(\mathbf{P})$

Perturbed simulation of the rider along the course

$T(\mathbf{P}+\Delta\mathbf{P})$
$g(\mathbf{P}+\Delta\mathbf{P})$

$\partial T/\partial\,\mathbf{P}$
$\partial g/\partial\,\mathbf{P}$

Modified power distribution

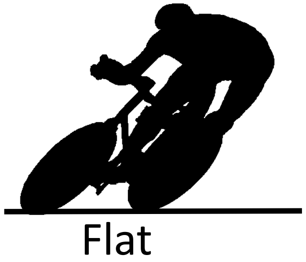
$P=P_{new}$

Optimization

$P=P_{opt}$

Optimized power distribution

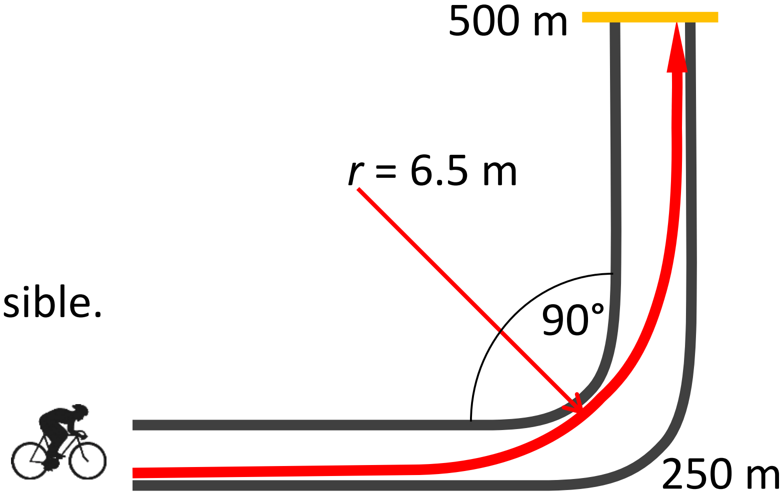
Simulated parameter settings



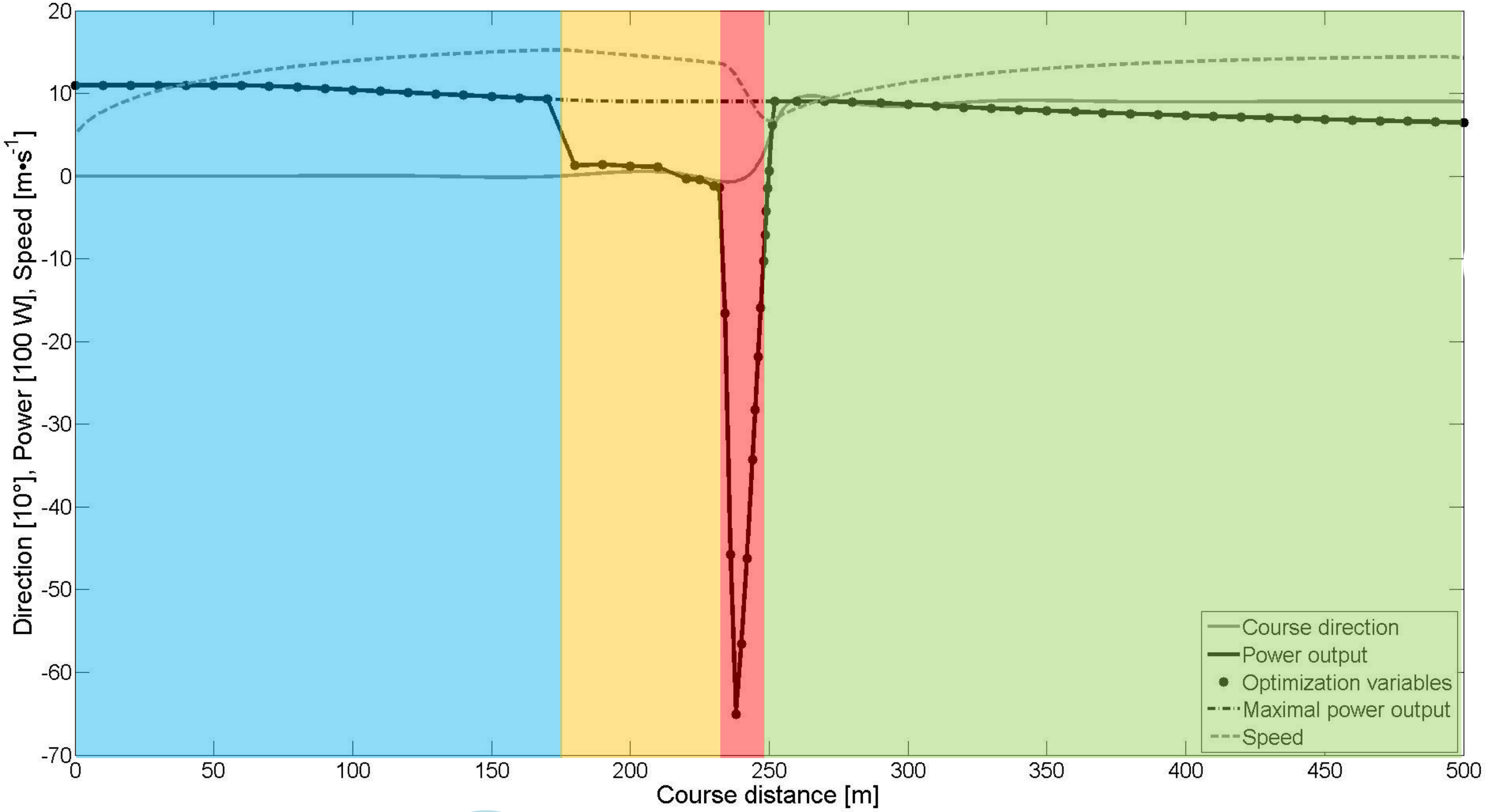
- Maximal power = 1100 W.
- Maximal braking power = 10 000 W.

- Static friction coefficient = 0.7.

Objective: Get to the finish line as quickly as possible.



Results



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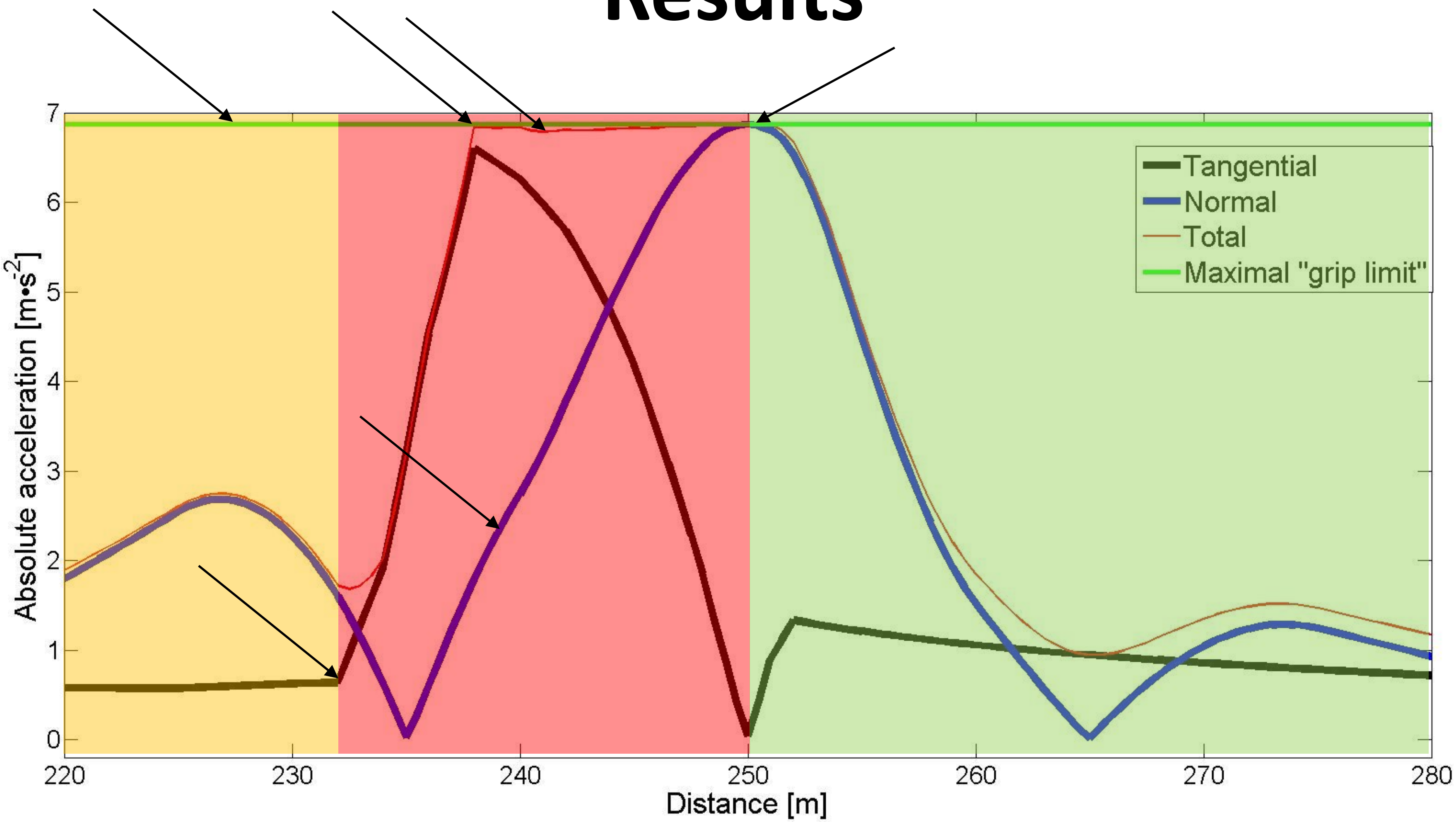
Results



Absolute acceleration [m•s^{-2}]
0
1
2
3
4
5
6
7
220
230
240
250
260
270
280
Distance [m]
Tangential
Normal
Total
Maximal "grip limit"

Results

Finishing time = 40.5 s

Braking time = 1.9 s

Average braking force = 197 N

Average propulsive force = 73 N

Average speed = 44 km·h^{-1}

Max speed = 55 km·h^{-1}

Average propulsive power = 829 W

Maximal braking force = 539 N

Maximal Propulsive force = 220 N

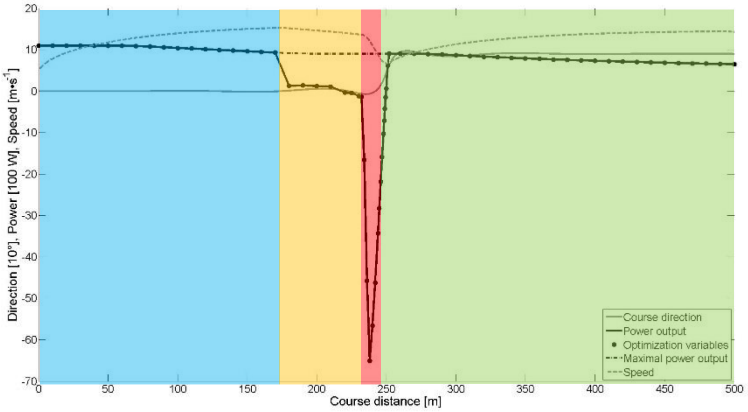
Total work = 31 kJ

Total braking energy = -5.7 kJ

Conclusions

Optimal pacing for sharp course bends includes 4 phases:

1. All-out acceleration.
2. Rolling (freewheeling).
3. Braking.
4. All-out acceleration.

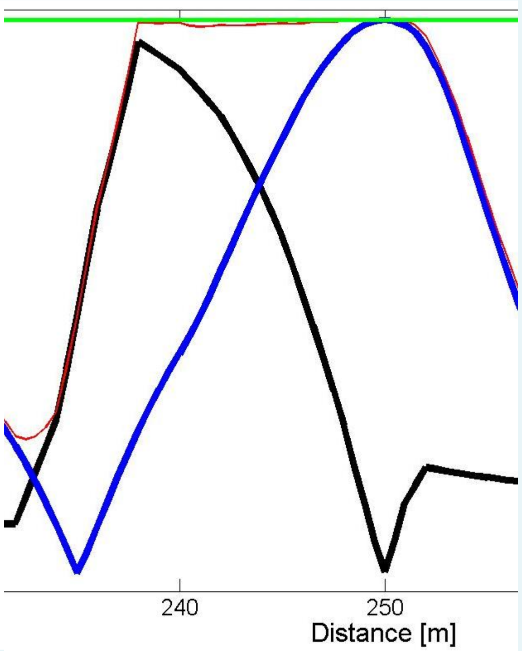


Conclusions

Optimal braking is:

executed directly preceding the apex of the corner, still

ensures the total acceleration is below the maximal allowable acceleration due to static friction.



Thank you